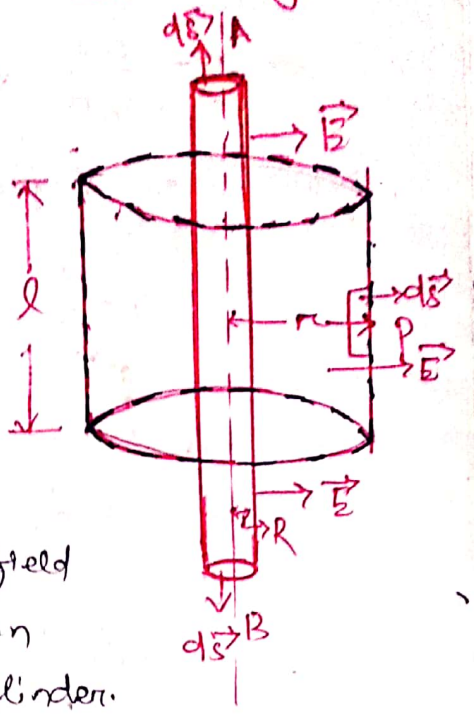
Electric field due to an infinite cylindrical charge:-



Consider an infinitely long cylinder of radius R and axis AB. Let σ be the charge density of the cylinder. Due to symmetry of the electric field, the field strength has same magnitude and direction is outward of the surface.

Let p be a point at a distance r from the axis AB. To calculate electric field at p we consider a cylindrical Gaussian surface s.t. pt. p lies on the surface of cylinder.

Flux due to the two ends of the surface is zero because electric field $\vec{E}$ and area vector $d\vec{S}$ are right angle to each other.

Hence the flux through the Gaussian surface is only due to the curved surface as Electric field $\vec{E}$ and area vector $d\vec{S}$ are along the same direction i.e. $\theta = 0^\circ$

$$\text{So, } \Phi_E = \oint \vec{E} \cdot d\vec{S}$$

$$= \oint E\,ds \cos 0^\circ$$

$$= E \oint ds \quad (\because E \text{ is constant})$$

$$= E(2\pi rl) \quad (\text{area of cylinder} = 2\pi rl)$$

But from Gauss's law,

$$\Phi_E = q/\epsilon_0$$

$$\Rightarrow E(2\pi rl) = q/\epsilon_0$$

$$\Rightarrow E = \frac{1}{2\pi\epsilon_0 l}\frac{q}{r} \quad -(1)$$

$$\text{As } \sigma = \frac{q}{2\pi la} \Rightarrow q = (2\pi la)\sigma$$

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$$\Rightarrow E = \frac{1}{2\pi\varepsilon_0 l} \frac{2\pi l a\sigma}{a\pi}$$

$$\Rightarrow E = \frac{a\sigma}{\varepsilon_0 r} \quad \text{---(2)}$$

Now charge per unit length of the cylinder

$$\lambda = \frac{q}{l} = \frac{2\pi a l \sigma}{l}$$

$$\Rightarrow \lambda = 2\pi a\sigma$$

$$\Rightarrow a\sigma = \frac{\lambda}{2\pi}$$

$\therefore$ Electric field due to a charged cylinder is,

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} \quad \text{---(3)}$$

Eqn (3) represents the electric field due to a line charge also. ie. the expression is same for both. Now for a charge lying inside the cylinder is zero because there is no charge inside the cylinder.

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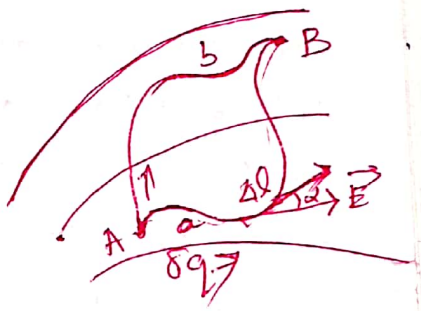
CONSERVATIVE NATURE OF ELECTROSTATIC FIELD:-

We know that the work is needed to be done when a charge is moved in an electric field from one point to another pt. in space. Also one can choose an indefinite no. of paths via which the charge is moved from one point to another. However it turns out that this work done in moving a charge from one pt. to another is path independent. This fact is due to the "conservative nature of electric field".

Let's consider an arbitrary electrostatic field produced by any arbitrary static charge distribution of charges. Let's calculate the work done in moving a test charge δq from a pt. A to another pt. B in this field.



For calculating the work done let's consider any arbitrary path AaB. To calculate work done during the movement of charge from A to B, we devide the path AaB into infinitesimally small elements. Let Δl be one such element and let α denote the angle b/w element Δl and electric field $\vec{E}$ then,

work done in movement of test charge over Δl is,

$$dW = (\delta q E)\, \Delta l \cos\alpha = \delta q (\vec{E} \cdot \Delta l) \quad \text{---(1)}$$

$\delta q E$ = electric force on δq

Similarly each element of the path AaB will contribute a term of form eq^n (1).

Hence, total work done by the electric forces in moving δq from A to B is, $W = \sum_A^B (\delta q E)\, \Delta l \cos\alpha = \sum_A^B \delta q (\vec{E} \cdot \Delta l)$ ---(2)

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As Δl is small elementary area or Δl is so small it can tends to zero, the summation in eqn (2) can be replaced by integral. Thus we have

$$W = \int_A^B \delta q (E \cdot dl) \quad \text{——— (3)}$$

Eqn (3) is known as the line integral of force $\delta q E$.

Let's assume that the workdone in moving test charge δq along b/w the two points is path independent. i.e. workdone is different for different paths. So, consider two paths AaB and BbA. and let the workdone in moving δq along AaB is greater than the work done along BbA. Then obviously δq moving along AaB and back along BbA will have non-zero, surplus value of energy at A. But according to law of conservation it is not possible. Since principle of conservation of energy can't be violated so our assumption of path dependence of the work done is wrong. Thus the work done along the paths AaB and BbA is same.

So, we conclude that the work done by electric forces are same along the paths which is connected b/w the points A and B. Thus we have,

$$W = \int_A^B \delta q (E \cdot dl) = \text{const.} \quad \text{— (4)}$$

The const. in eqn (4) indicates that workdone is path independent. which proves the independence nature of line integral and the conservative nature of electric field.

From eqn (4) now if the path under consideration is a closed one, then workdone by electric forces in moving a test charge around it is,

$$W = \delta q \oint E \cdot dl = 0 \quad \text{— (5)}$$

Again as δq is arbitrarily choosen we have,

$$\boxed{\oint E \cdot dl = 0} \quad \text{— (6)}$$

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This eqn (6) is valid for any real electrostatic field and is an equivalent statement of the conservative nature of the electric field. The integral $\oint E.dl$ is known as "circulation of the vector E".

ELECTRIC POTENTIAL

Consider an external agent which is used to moving a test charge δq from point A to B in an arbitrary electrostatic field. This external agent will work against the field. Let no kinetic energy is given to the charge, then work done by the external agent can be thought of as the change in potential energy of the charge in going from one pt. to another. Thus the work done by the external agent can be expressed as equal and opposite to the work of electric forces and is given by,

$$W_{ext} = -\int_A^B \delta q (E \cdot dl) \quad \text{---(1)}$$

Eqn (1) interms of the change in the electric potential energy in going from A to B. Thus,

$$(P.E)_B - (P.E)_A = -\int_A^B \delta q (E \cdot dl) \quad \text{---(2)}$$

Eqn (2) is the general expression for the diff b/w the potential energies of the test charge δq at points A & B. in an electric field.

However as we can't define P.E at any point, we can define only the P.E. of a charge at a given point w.r.t some arbitrary reference point where the P.E is considered to be zero. From mathematical pt. of view the reference pt. is choosen to be an ∞. Thus "the" potential energy of the test charge is zero when it is far removed from the charge setting the field."

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With this assumption of the choice of the reference point as ∞ considering the charge to have no initial kinetic energy, the potential energy of the system is defined as the work required to bring a charge from an infinite distance. Thus, in eqn (2) let the pt. A be situated at ∞ so that $(P.E)_A = 0$ and it is given by,

$$\boxed{(P.E)_B = -\int_{\infty}^{B} \delta q (E \cdot dl)} \quad \text{--- (3)}$$

If we devide eqn (3) by δq, we get the potential energy per unit charge at some point B with infinity as the reference point for potential energy. Thus we have,

$$\boxed{\frac{(P.E)_B}{\delta q} = -\int_{\infty}^{B} E \cdot dl} \quad \text{--- (4)}$$

Eq (4) serves a useful purpose in dealing with the potential energy of a test charge at a pt. or the difference in the potential energies. This eqn (4) has a unique value for each pt. B since it is path independent.

As the integral represents the work per unit charge done by an external agent in bringing the unit charge from ∞ to the consideration pt.. Therefore, we can calculate how much work is done by an external agent in moving the test charge from ∞ to another pt. i.e. P.E per unit charge. So, one can describe the static electric field in terms of the scalar quantity.

Let V = P.E per unit charge

$$\Rightarrow \boxed{V = -\int_{\infty}^{B} E \cdot dl} \quad \text{--- (5)}$$

Eqn (5) is reffered as the electrostatic potential at the field pt. under consideration w.r.t ∞ as the reference point.

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Thus physically we define "the potential at a pt. in any electrostatic field as the work per unit charge necessary to bring any +ve charge from infinity to that point".

$$V = \frac{W}{q_0}$$

* Unit of electric potential is $1\,V = \frac{1\ \text{Joule}}{1\ \text{coulomb}}$

Potential Difference

The potential as defined in eqn (5) is not an absolute entity because of the arbitariness involved in the choice of the reference point. It is therefore more realistic to talk of the difference b/w the potentials of two points of an electric field. The potential difference is more fundamental entity since it determines change in potential energy of a charge when it moves from one point to another in an electrostatic field. Thus we often prefer to talk of potential difference rather than potential itself

Deviding eqn (2) by δq we have,

$$\frac{(P\cdot E)_B - (P\cdot E)_A}{\delta q} = -\int_A^B E\cdot dl$$

$$\Rightarrow \frac{(P\cdot E)_B}{\delta q} - \frac{(P\cdot E)_A}{\delta q} = -\int_A^B E\cdot dl$$

$$\Rightarrow \boxed{V_B - V_A = -\int_A^B E\cdot dl}$$

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